Problem 2.8

A connecting rod weighing 21.35 N oscillates 53 times in 1 min when suspended as shown in Fig. P2.8. Determine its moment of inertia about its center of gravity, which is located 0.254 m from the point of support.

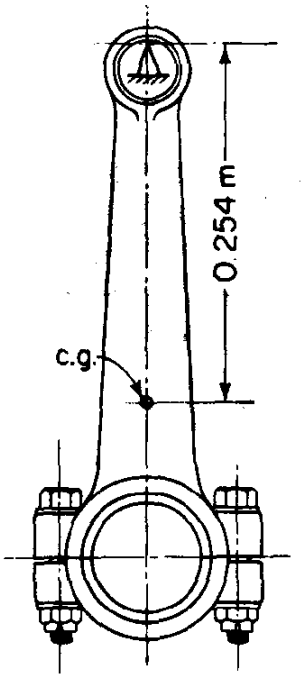


FIGURE P2.8.

Solution

If the rod is displaced some angle and allowed to oscillate, then it behaves as a physical pendulum. Apply the rotational analog of Newton's second law in order to obtain the equation of motion.

$$\sum \boldsymbol{\tau} = J\boldsymbol{\alpha}$$

Here $\boldsymbol{\tau}$ is the external torque, J is the mass moment of inertia, and $\boldsymbol{\alpha}$ is the angular acceleration. Consider the sum of the torques about point O, the chosen origin.

$$\mathbf{r}_{\text{cg}} \times \mathbf{w} = J_O\boldsymbol{\alpha}$$

$$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ L\sin\theta & -L\cos\theta & 0 \\ 0 & -w & 0 \end{vmatrix} = J_O\alpha\hat{\mathbf{z}}$$

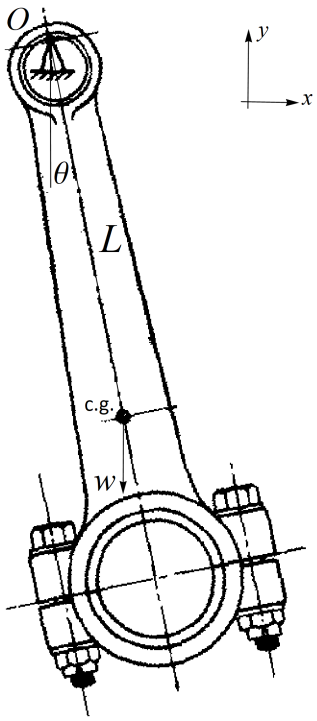
Evaluate the cross product.

$$-wL\sin\theta\hat{\mathbf{z}} = J_O\ddot{\theta}\hat{\mathbf{z}}$$

The components must then be equal.

$$-wL\sin\theta = J_O\ddot{\theta}$$

$$\ddot{\theta} = -\frac{wL}{J_O}\sin\theta$$



The motion is simple harmonic with the assumption that θ is small: $\sin\theta \approx \theta$.

$$\ddot{\theta} = -\frac{wL}{J_O}\theta$$

As a result, the angular frequency of oscillation is

$$\omega = \sqrt{\frac{wL}{J_O}}.$$

Write it in terms of the linear frequency f and solve for J_O.

$$2\pi f = \sqrt{\frac{wL}{J_O}}$$

$$J_O = \frac{wL}{4\pi^2 f^2}$$

Apply the parallel-axis theorem,

$$J_O = J_{\text{cg}} + \frac{w}{g}L^2,$$

in order to obtain the moment of inertia about a parallel axis through the rod's center of gravity.

$$J_{\text{cg}} + \frac{w}{g}L^2 = \frac{wL}{4\pi^2 f^2}$$

Therefore, noting that $1\ \text{N} = 1\ \text{kg}\cdot\text{m/s}^2$,

$$\begin{aligned} J_{\text{cg}} &= \frac{wL}{4\pi^2 f^2} - \frac{w}{g}L^2 \\ &= \frac{(21.35\ \text{N})(0.254\ \text{m})}{4\pi^2\left(53\ \frac{\text{cycles}}{\cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\ \text{s}}\right)^2} - \frac{21.35\ \text{N}}{9.81\ \frac{\text{m}}{\text{s}^2}}(0.254\ \text{m})^2 \\ &\approx 0.0356\ \text{kg}\cdot\text{m}^2. \end{aligned}$$